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Measuring the Effectiveness of Contemporary Models to Forecast Market Risk of Commercial Banks in Bangladesh

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Abstract

This paper investigates the validity of Hybrid model in forecasting market risk considering VaR (Value at Risk) recommended by (Boudoukh, Richardson and Whitelaw, 2007). It then considers the coherent version of the risk measure VaR, ES (Expected shortfall) in forecasting daily investment risk for six commercial banks operating in Bangladesh for the first time applying the Hybrid Model. We consider the daily log return covering from 2010 to 2020, a particularly idiosyncratic performance period for Dhaka Stock Exchange (DSE). We have adopted RiskMetrics™(RM) and Historical Simulation (HS) models as less complicated alternate to Hybrid model, which is a combination of HS and RM model. We then consider GARCH (generalized autoregressive conditional heteroscedasticity) and GARCH-t models being more complicated alternate to the Hybrid model. Our investigation reveals that Hybrid model, being a parsimonious model, performs poorly compared to GARCH and GARCH-t models in case of VaR forecast and it also performs poorly compared to RM, HS, GARCH and GARCH-t models in case of ES forecasts as evidenced by the outcomes of several back-tests as per BASEL-III accord.

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1. Introduction

Risk can be measured or estimated using the process of mapping random variables representing profit and loss (P/L) into real numbers revealing the amount of capital required to defend against insolvency. In financial institutions especially in banks, it's usual to predict risk with probability distributions and divulge risk in terms of scalar-valued risk measures (see Kratz M., Lok Yen & McNeil j. (2016)). If the random variable X representing the P&L (profit and loss) position follows the normal distribution with mean μ and variance σ^2, the f(x) obeys the following probability function:

$$\boldsymbol{f(x)} = \frac{\mathbf{1}}{(\boldsymbol{\sigma}\sqrt{\mathbf{2}\boldsymbol{\pi}})}\mathbf{exp}\left[-\frac{\mathbf{1}}{\mathbf{2}}\left(\frac{\boldsymbol{x}-\mu}{\boldsymbol{\sigma}}\right)^{\mathbf{2}}\right] \ldots\ldots(\mathbf{1})$$

where f(x) is following normal probability function (pdf) with mean 0 and standard deviation 1. The most common and recent approach in measuring market risk is the application of VaR (value at risk) model. RM approach developed by JP Morgan in 1994 followed by publishing the detailed methodology in 1996 adopted by BASEL committee for banking supervision in

1998 recommended all of its member banks to maintain capital reserve on the basis of estimation of VaR. Value at Risk (VaR) is defined as the maximum loss on a certain confidence level (α) with specific time horizon (t) as follows:

$$\boldsymbol{VaR_{t,\alpha}(X)} = \mathbf{inf}\{\boldsymbol{x}|\mathrm{P}(\mathrm{X}_t < -\mathrm{x}) \leq \alpha\} \ldots\ldots(\mathbf{2})$$

where X_t is the net value of an asset or portfolio after n days defined as $X_t = S_t - S_{t-1}$. The loss from this can be defined as $L_t = -X_t$

Usually a number of approaches constitute the VaR measures including historical simulation method where past or historical data is used to estimate the quantile not requiring any explicit assumption regarding the shape of the return distribution. In contrast, it's criticized due to utilizing past data in order to describe current return whether or not the data is weighted may be affected by any change in underlying market condition (Haung y. Alex, 2010). In addition, Hendricks (1996) identified that in historical simulation, larger sample diminishes the variability of VaR estimates where VaR measure is imprecise while using short sampling period. Another popular estimate of VaR measures is Monte Carlo (MC) simulation that spawns stochastic forecasted price paths to imprecise the movements of financial assets. In VaR methods, the desired quantile can be directly attained from these random or stochastic paths (Haung y. Alex, 2010). This MC simulation has become a powerful tool for pricing financial assets due to its flexibility accommodating a wide range of characteristics. However, another common method is parametric estimation including variance-covariance method that forecasts VaR using volatilities of financial assets estimated from past returns. Moreover, EWMA standing for Exponential Weighted Moving Average model developed by RM method incorporates more weights to recent observations while calculating volatility from historical returns. In addition, GARCH (generalized autoregressive conditional heteroscedasticity) originated by Bollerslev (1986) has also attempted to forecast accurate volatility for estimating VaR measures using parametric models assuming an explicit form of the return distribution which is the major drawback of this parametric estimation as the distribution of return is rarely normal (Hull & White, 1998). Moreover, the distribution of different financial assets is probably having different shapes such as student's t-distribution, a stable paretian distribution (Mittnik, Paolella & Rachev, 2002), a fusion of normal distribution (Venkataraman, 1997) or the generalized error distribution (Rossello, 2008). Many researchers also found that VaR measures based on normal or student's t-distribution underestimate variance and are subject to upward bias because return distributions are fat-tailed (Longin, 1996). Although this distributional limitation has been resolved in two recent developments of VaR measures including EVT (extreme value theory) and quantile regression where EVT improves the parametric estimation of VaR by not attempting to model the whole distribution; instead it models only the tails of the return distribution (Haung y. Alex, 2010), this EVT approach has been also criticized for suffering the problem of difficulty in precisely modeling the tail distribution of return (McNeil, 1997; Neftci, 2000). The quantile regression, second recent development in VaR measures, originated by Engle & Manganelli (2004) to assess a progression of quantiles directly from the data rather than modeling the distribution of returns although this has been criticized for underestimating the real level of risk (Kuester, Mittnik, & Paolella, 2006). Apart from this, the copula approach (Malevergne & Sornette 2003, Rodriguez 2004, Caillault & Guegan 2005), another widely used tool of VaR measures has been also criticized of non-stationarity of the bivariate distribution function of a two-dimension portfolio.

Although VaR complies with the three major properties of risk measures such as Normalization, Translation invariance and Monotonicity (Johan Engvall 2016, Dowd Kevin 2005), Following drawback of not satisfying subadditivity condition of VaR measure along with failure of capturing the tail (extreme) risk (Artzner et al., 1999; Accerbe and Tasche, 2002) converting it into a non-coherent risk measure accelerates financial institutions to use expected shortfall (ES) being coherent risk measure having subadditivity property for estimating market risk component considering certain time horizon with confidence level on a portfolio. As a consequence, The BASEL committee of Banking Supervision has replaced Expected Shortfall (ES) with VaR measures to estimate the market risk component of financial institution. In contrast, ES can't be attained as the unique minimizer of the expected loss function (Gneiting, 2011) although Fisher and Ziegel (2016) revealed that both VaR and ES are jointly elicitable that further navigates evaluating the ES jointly with VaR in a unified framework (Fissler et al., 2016).

This paper has adopted five different VaR measures in a rolling forecast basis including Historical Simulation, Weighted Historical simulation or Hybrid approach, GARCH, GARCHt (Taylor 2019; Patton et al., 2006) and RM approach followed by Expected Shortfall (ES) measures considering these same approaches revealing proxy for quantifying market risk. ES can be defined as follows:

$$\boldsymbol{ES_{\propto}(X)} = \frac{\mathbf{1}}{\propto}\int_{\mathbf{0}}^{\propto} \boldsymbol{VaR_u(X)du} \ldots\ldots (\mathbf{3})$$

Where, if X has a continues distribution, it can also be rewritten as follows:

$$\boldsymbol{ES_{\propto}(X)} = \mathbf{E[L|}\boldsymbol{VaR_{\propto}}\mathbf{\alpha(X) \leq\ L]} \ldots\ldots (\mathbf{4})$$

Alternatively, ES can be also defined as the expected value of loss in excess of VaR of an asset or portfolio of assets as follows:

$$\boldsymbol{ES_{\propto}(X)} = \mathbf{E[L|L >} \boldsymbol{VaR_{\propto}}\mathbf{\alpha(X)\ ]} \ldots\ldots (\mathbf{5})$$

The next segment consists of data and methods followed by empirical analysis with discussion and concluding remarks.

2. Data and Methods

This is an empirical investigation on quantifying the risk of financial position using probability distribution depending on the tools mapping random variables into numbers that are known as risk measures being understood as a means of providing risk evaluation in forms of capital amount which is required to defend any future unexpected loss. In this paper, we have adopted several models of value at risk (VaR) and Expected Shortfall (ES) measures revealing proxy for quantifying market risk based on the earlier investigations on these estimates and the motivation behind the objectives mentioned in previous section. The research design of this investigation has been described below:

2.1 Sample size For accomplishing this paper, we have collected almost

2,500 observations of daily closing share price of each bank to calculate the daily log returns of commercial banks listed in Dhaka Stock Exchange (DSE) from January 2010 to June 2020. Hence, we have decided to include six leading listed commercial banks for this investigation.

2.2 Data Collection Procedures we have adopted secondary sources of data collected from January 2010 to June 2020 in DSE. This volume of data has been coded in MS excel and then driven to software such as Matlab 2020^R to execute further steps of conducting data processing and analysis.

2.3 Data Analysis Mechanism

Estimating VaR and ES for six commercial banks listed in Dhaka Stock Exchange considering daily closing share price data of each bank from January 2010 to June 2020 requires the adoption of following models:

a. Historical Simulation (HS) model The choice of this model is justified due to simplest non-parametric methods to forecast VaR determined by its α quantile of the estimated distribution of log-returns from our time series observation over a chosen period asking for a risk measure that needs a distribution with more probability mass in the tails as depicted below:

$$\boldsymbol{VaR_{\alpha}(t) = q_n \dots\dots (6)}$$

b. RiskMetrics™ (RM) model Unlike the earlier method, this method doesn't assign equal weights to the past returns. Instead, it assigns unequal weights which is exponentially decreasing trend showing higher weights for the most recent returns as they pursue current returns more heavily compared to previous returns of asset as revealed below:

$$\boldsymbol{\widehat{\sigma_t^2} = \frac{1}{c}\sum_{i=1}^{w_e} \lambda^{i-1} y_{t-i}^2 \dots\dots (7)}$$

Where c is a normalizing constant followed by notation:

$$c = \sum_{i=1}^{w_e} \lambda^{i-1} = \frac{1-\lambda^{w_e}}{1-\lambda}$$

And λ, in practice, is the decay factor showing relative importance of past data observing a value of 0.94

c. Weighted Historical Method or Hybrid model This approach incorporates two methods by estimating the percentiles of return directly considering diminishing weights on past or historical data. It initiates with ordering the returns over the observation period according to Historical Simulation or HS method followed by attributing exponentially declining weights to historical returns like RiskMetrics™ method although HS approach assigns equal weights to each return in building the conditional empirical distribution. As a consequence, while obtaining the 2.5% VaR using 1230 data set or returns involves identifying the 31st lowest observation in HS approach, it may involve more or less observations in the Hybrid approach as per following steps (Boudoukh, Richardson and Whitelaw, 2007):

Step 01: Let r is the realized return from t-1 to t. To each of the most recent K returns: $r_{(t)}$, $r_{(t-1)}$, ..., $r_{(t-k+1)}$, assign a weight such as $[(1-\lambda)/(1-\lambda^k)]$, $[(1-\lambda)/(1-\lambda^k)]\lambda$,..... $[(1-\lambda)/(1-\lambda^k)]\lambda^{k-1}$, respectively considering the weights sum to 1.

Step 02: Rearrange the returns in ascending order.

Step 03: In order to obtain the x% VaR, initiate from the lowest return and keep accumulating the weights until x% is achieved subject to the application of Linear interpolation to achieve exactly x% of the distribution.

It's to be noted that the dynamic VaR estimation provides an estimate of the x% VaR for the sample period. So, the probability of observing a return less than the estimated VaR would be x% (Boudoukh, Richardson and Whitelaw, 2007):

$$prob[r_{t+1} < -VaR_t] = x\%.$$

d. GARCH and GARCH-t model Generalized Autoregressive Conditional Heterosckedasticity being an econometric model of volatility dynamics provides VaR estimate reflecting current volatility background that believes the fact that volatility (σ) is not always constant so that we need to implement the parsimonious but efficient GARCH (1,1) model providing a good fit of data and making it less probable to violate the non-negativity constraints as depicted below:

$$\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \dots\dots (8)$$

$$\text{where } u_t = \sigma_t . \epsilon_t,$$

ϵ_t =normal and random variable independently and identically dsitributed

ϵ_t = Student´ s t random variable independently and identically dsitributed

This GARCH model can be fitted by maximum likelihood (ML) method given the conditional standard deviation that will be used to estimate VaR under the assumption of normal distribution as well as t-distribution as follows:

$$\hat{r}_t = \mu + u_t$$

$$VaR_{97.5\%t} = -\hat{r}_t - 1.96\sigma_t \dots\dots (9)$$

where σ_t=historical GARCH forecast of volatility of return of asset i on day t made at the end of t-1

$\hat{r}_t$ = scaled return

Now we can compute 97.5% VaR measure for the said observations considering GARCH and GARCH-t models where innovations are assumed to be followed under normal and t-distribution respectively.

Following the estimations of VaR of these 5 banks at 97.5% level under BASEL-III accord, the expected shortfall (ES) can also be estimated considering these five above models. As ES is defined as an integral of VaR according to equation number 2 mentioned above, this integration can be approximated as a sum of different VaR levels given below:

$$ES_\propto(X) = \frac{1}{\propto}\int_0^\propto VaR_u(X)du \approx \frac{1}{N}\sum_{k=1}^{N} VaR_{\frac{k\propto}{N}}(X) \dots\dots (10)$$

The approximation suggested by Emmer, Kratz and Tasche (2015)

$$ES_\propto = \frac{1}{4}[VaR_\alpha(X) + VaR_{0.75\alpha}(X) + VaR_{0.50\alpha}(X) + VaR_{0.25\alpha}(X)] \dots\dots (11)$$

However, Johan Engvall (2018) applied the following approximation of integration to be beneficial when used on VaR estimated with HS and HYB method as described earlier:

$$ES_{\propto} = \frac{1}{5}\begin{bmatrix} \mathrm{VaR}_{\alpha}(X) + \mathrm{VaR}_{0.80\alpha}(X) + \\ \mathrm{VaR}_{0.60\alpha}(X) + \mathrm{VaR}_{0.40\alpha}(X) \\ +\mathrm{VaR}_{0.20\alpha}(X) \end{bmatrix} \ldots\ldots(12)$$

After estimating VaR and ES for each of these 05 commercial banks using the models mentioned, we will adopt Back-testing procedure consisting of a set of statistical procedures aimed at inspecting whether real losses are in compliance what has been predicted or estimated. Following three Back-testing methods will be applied to compare the performance of VaR measures of banks:

a. POF-test This test statistic also known as Kupiec test will be adopted to find out whether there is a large difference between observed failure rate and theoretical failure rate as constructed below:

$$LR_{uc} = -2\ln\frac{(1-P)^{T-x}p^{x}}{\left[1-\left(\frac{x}{T}\right)\right]^{T-x}\left(\frac{x}{T}\right)^{x}} \ldots\ldots(13)$$

where p=failure rate and T=testing period

b. Christoffersen's Interval forecast test Following test statistic will be applied to test the independence hypothesis where Ho = $r_{01} = r_{11}$

$$LR_{ind} = -2\ln\frac{(1-\pi)^{T00+T10}\pi^{T00+T11}}{[1-\pi_{01}]^{T00}\pi_{01}^{T01}(1-\pi_{11})^{T10}\pi_{11}^{T11}} \ldots\ldots\ldots(14)$$

where π_{01} is the probability of having a violation tomorrow given that today has no violation π_{11} is the probability of tomorrow being a violation given today is also a violation;

This test statistic follows chi-squared distribution with one degree of freedom,

$$LR_{ind} \sim \chi 2$$

c. BASEL Traffic Light Approach This method depends on the excess ratio followed by notation α expressed as:

$$\hat{\alpha} = \frac{1}{N}\sum_{t=1}^{N} I_t \ldots\ldots\ldots(15)$$

The responses of this model have been classified into three categories such as Green Zone, Yellow Zone and Red Zone showing there is no problem, potential problem and strict problem respectively with predictive accuracy of the model considering following formula:

$$F(I_t) = \sum_{k=0}^{I_t}\binom{N}{K} p^k(1-p)^{N-k} = \alpha\begin{Bmatrix} \text{Red zone: } \alpha \geq 0.999 \\ \text{Yellow zone } \alpha \geq 0.975 \\ \text{Green zone } \alpha \geq 0.950 \end{Bmatrix} \ldots\ldots\ldots(16)$$

Apart from these approaches, other back-testing procedures will be implemented to validate the VaR models estimating market risk observing the daily log return from closing share price of each listed banks in DSE since 2010 in Bangladesh.

Following unconditional test statistic of back testing procedure has been applied for Expected shortfall (ES) estimations:

$$Z_{unconditional} = \left(\frac{1}{N_{pVaR}}\right)\sum_{t=1}^{N}\frac{X_t I_t}{ES_t} + 1 \ldots\ldots\ldots(17)$$

Where, N is the number of time periods in the test window. X_t is the portfolio outcome, that is, the portfolio return or portfolio profit and loss for period t. pVaR is the probability of VaR failure defined as 1-VaR level. ES_t is the estimated expected shortfall for period t. I_t is the VaR failure indicator on period t with a value of 1 if $X_t < -\mathrm{VaR}_t$, and 0 otherwise.

This test statistic usually has an expected value of 0 and it produces negative value when the risk of underestimation happens. The critical values are required to decide how negative it (test statistic value) should be to reject the model. In addition, this critical value can be determined based on distributional assumptions for the corresponding outcomes of X_t being proxy for showing portfolio return or portfolio profit and loss for period *t. ES (expected shortfall) Back test consists of two sets of critical value table. The first set of table assumes that X_t follows standard normal distribution for executing unconditional Normal test whereas the second critical value table assumes that X_t follows t-distribution for executing unconditional-T test.*

3. Empirical results with discussion

Adopting the five different models of estimating VaR and ES risk measures for the selected commercial banks, we use the dataset of daily closing share price chosen as a representative set of 'interesting' economic series because of sustaining a priori that high order moments depending on skewness as well as moments reveal different degrees of challenge in estimation of VaR and ES. The data series covers from 2010 to 2020 for Islami Bank Bangladesh Ltd (IBBL), Al-Arafa Islami Bank Ltd, Bank Asia Ltd, The City Bank Ltd, Prime Bank Ltd (PBL) and Mutual Trust Bank (MTBL) Ltd.

We have adopted a 250-day trading window throughout this calculation of VaR and ES considering 2460 returns data of each bank where 1230 returns out of these 2460 observations are used as back data or in-sample data. The average statistics consists of 1230 data points with 30.75 tail events expected in 2.5% of the tail as per BASEL accord-III.

The following figure is representing the estimations of VaR (Value at Risk) for IBBL, Al-Arafa Bank, Bank Asia, The City Bank, Prime Bank and MTBL considering five approaches such as Historical simulation (HS), Weighted historical simulation (WHS) or Hybrid, GARCH, GARCH with t-distribution, RiskMetrics™ (RM) approach as described earlier in methodology segment of this paper. The parameter for RM and Hybrid (WHS) model is assumed to be 0.94 and 0.98 followed by notation of lamda and gamma respectively in our VaR forecasts for out sampled observations. The comparative scenario of the estimation reveals that the forecast of VaR measures is fitted with five different models including HS, Weighted HS or Hybrid, GARCH, GARCH-t and RM depending on the earlier or back data. The VaR forecasts are well fitted compared to the actual returns divulged as P/L in the following figure. The comparative scenario of the estimation reveals that the forecast of VaR measures is fitted with five different approaches including HS, Weighted HS or Hybrid, GARCH, GARCH-t and RM depending on the earlier or back data. The VaR forecasts are well fitted compared to the actual returns divulged as P/L in the following figures.

Fig 1: Estimation of VaR for IBBL

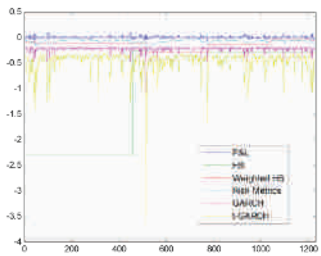


Fig 2: Estimation of VaR for Al-Arafa Bank

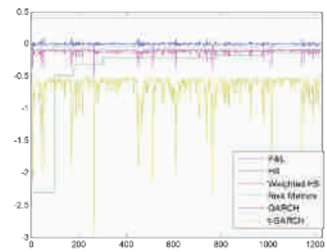


Fig 3: Estimation of VaR for Bank Asia

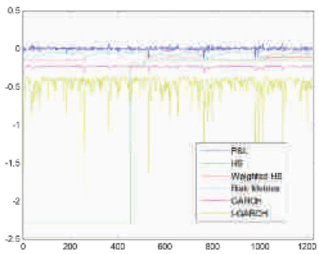


Fig 4: Estimation of VaR for The City Bank

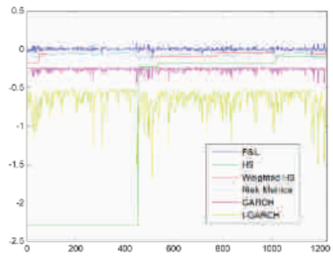


Fig 5: Estimation of VaR for Prime Bank

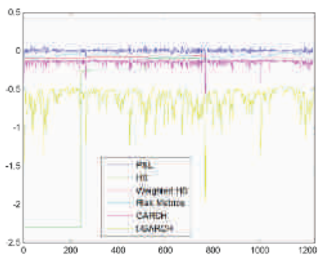
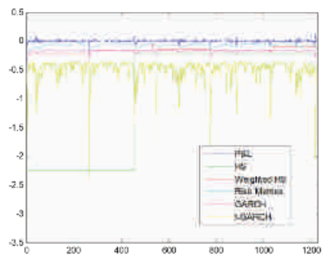


Fig 6: Estimation of VaR for MTBL



Note: The above figures consist of six subplots, each representing the estimation of VaR forecasts for different banks: IBBL, Al-Arafa Bank, Bank Asia, The City Bank, Prime Bank, and MTBL. The models employed are the Historical Simulation (HS), Weighted Historical Simulation (Hybrid model), Risk Metrics, GARCH, and GARCH-t. The Profit and Loss (P&L) series for out-sample period is also plotted for reference.

As reflected in the above figures, for IBBL, The models produce diverse VaR forecasts, with the GARCH-t and GARCH models showing significant adjustments during periods of market stress. The Risk Metrics model displays a stable forecast but underestimates the risk compared to GARCH-based models. Like IBBL, the GARCH-t model estimates show pronounced sensitivity, with high peaks during volatile periods, which may be observed in the P&L deviations for Al-Arafa Bank.

For Bank Asia and The City Bank PLC, These figures show comparable behaviors where the Risk Metrics model maintains a smooth trajectory, while GARCH and GARCH-t forecasts align more closely with market stress points indicated by the P&L line.

For Both Prime Bank and MTBL, The GARCH and GARCH-t models provide more reactive and sometimes significantly higher VaR forecasts, especially in periods that align with notable losses in the P&L data, suggesting that these models capture risk peaks more effectively. Precisely, The GARCH-t model consistently shows the highest and most volatile VaR estimates due to its heavier tails, making it effective at capturing extreme risk during volatile periods for the banks.

Moreover, observing the VaR forecasts for out sample of 1230 observations estimated with all these-five models, there are only 1, 3, 5, 0 and 0 violations found for IBBL followed by 0, 2, 5, 0 and 0 violations found for Al-Arafa Bank. In addition, there are 0, 3, 8, 0 and 0 violations found for Bank Asia Ltd followed by 0, 4, 2, 0 and 0 violations found for The City Bank Ltd and 0, 3, 7, 0 and 0 violations for MTBL. Moreover, there are 1, 5, 5, 0 and 0 violations for Prime Bank Ltd in the last estimation out of 100 times estimations of VaR forecasts in HS, Hybrid, RM, GARCH, GARCH-t model respectively compared to out-sampled P&L. Considering these less number of violations over the out-sample period, all these five models estimating VaRs for these six banks have performed well as our expected number of violations is 30.75 which will be further tested to check the validity and performance of these models using Back-testing approaches:

Table 1: Summary of Back-testing of all five VaR models of four Banks

VaR Models	VaR Level	out-sample's observations	Expected Failures	Observed failures					
				PBL	IBBL	MTBL	Al-Arafa	CBL	Bank Asia
HS	0.975	1230	30.75	01	01	00	00	00	00
HYB	0.975	1230	30.75	05	03	03	02	04	03
GARCH	0.975	1230	30.75	00	00	00	00	00	00
GARCH-t	0.975	1230	30.75	00	00	00	00	00	00
RM	0.975	1230	30.75	05	05	07	05	02	08

Source: Authors' estimations based on Matlab 2020a

In addition, the outcomes of back-testing procedure reveal that all of these five models estimating VaR forecasts are fallen under the green zone as the number of violations or exceptions from VaR back-testing during the previous 250 trading days is less than 5 (BCBS, 2019) according to Traffic light approach recommended in BASEL-III accord. The results of other back-testing approaches described in methodology segment have been mentioned below to check the performance of all these five models applied to forecast VaR measures of all six banks:

Table 2: Outcomes of Back-testing of all five models estimating VaR forecast for Banks

Bank ID	VaR Models	VaR Level	TL	POF	CCI	TUFF
PBL	HS	0.975	Green	reject	Accept	reject
	HYB		Green	reject	Accept	reject
	GARCH		Green	reject	Accept	reject
	GARCH-t		Green	reject	Accept	reject
	RM		Green	reject	Accept	reject
CBL	HS	0.975	Green	reject	Accept	reject
	HYB		Green	reject	Accept	Accept
	GARCH		Green	reject	Accept	reject
	GARCH-t		Green	reject	Accept	reject
	RM		Green	reject	Accept	Accept
MTBL	HS	0.975	Green	reject	Accept	reject
	HYB		Green	reject	Accept	Accept
	GARCH		Green	reject	Accept	reject
	GARCH-t		Green	reject	Accept	reject
	RM		Green	reject	Accept	Accept
IBBL	HS	0.975	Green	reject	Accept	reject
	HYB		Green	reject	Accept	Accept
	GARCH		Green	reject	Accept	reject
	GARCH-t		Green	reject	Accept	reject
	RM		Green	reject	Accept	reject
Al-Arafa	HS	0.975	Green	reject	Accept	reject
	HYB		Green	reject	Accept	reject
	GARCH		Green	reject	Accept	reject
	GARCH-t		Green	reject	Accept	reject
	RM		Green	reject	Accept	reject
Bank Asia	HS	0.975	Green	reject	Accept	reject
	HYB		Green	reject	Accept	Accept
	GARCH		Green	reject	Accept	reject
	GARCH-t		Green	reject	Accept	reject
	RM		Green	reject	Accept	Accept

Note: TL = Traffic Light, PoF= proportion of failure, CCI= Christoffersen's Interval, TUFF= Time until first failure

Source: authors' estimations based on Matlab (2020a) Output

Although the POF test rejects all models due to the rejection of the assumption regarding the discrepancy between observed failure rate and theoretical failure rate, the Christoffersen's Interval (CCI) back-test accepts all models considering the test statistic that follows a chi-squared distribution with one degree of freedom to test the independence of hypothesis. In addition, TUFF test statistic rejects all models except Hybrid and RiskMetrics™ model forecasting VaR measures for CBL, MTBL, IBBL and Bank Asia Ltd respectively but it rejects all models for PBL and Al-Arafa Bank Ltd respectively.

It's mentionable that adopting 2.5% alpha as per BASEL capital accord III requires 100 times rolling estimation of VaR forecasts for 1230 out-sampled daily P&L considering all these five models using 1230 in sample observations. These rolling estimates of VaR measures have been passed through a process of back-testing approaches each time that depicts the outcomes in a relative form as reported below:

Table 3: Relative outcomes of 100 times Back-testing of all five VaR models for all banks

Bank ID	Back-testing Approaches	Outcomes	VaR Models				
			HS	HYB	GARCH	GARCH-t	RM
PBL	TL (Traffic Light)	Green	100%	99%	100%	100%	100%
		Yellow	0%	1%	0%	0%	0%
		Red	0%	0%	0%	0%	7%
	PoF (Proportion of Failure)	Accept	0%	4%	0%	0%	3%
		Reject	100%	97%	100%	100%	97%
	CCI (Christoffersen's Interval)	Accept	100%	98%	100%	100%	100%
		Reject	0%	2%	0%	0%	0%
	TUFF (Time until First Failure)	Accept	0%	0%	0%	0%	0%
		Reject	100%	100%	100%	100%	100%
Al-Arafa	TL (Traffic Light)	Green	100%	100%	100%	100%	100%
		Yellow	0%	0%	0%	0%	0%
		Red	0%	0%	0%	0%	0%
	PoF (Proportion of Failure)	Accept	2%	4%	0%	0%	0%
		Reject	98%	96%	100%	100%	100%
	CCI (Christoffersen's Interval)	Accept	97%	96%	100%	100%	100%
		Reject	3%	4%	0%	0%	0%
	TUFF (Time until First Failure)	Accept	4%	8%	0%	0%	9%
		Reject	96%	92%	100%	100%	91%

MTBL	TL (Traffic Light)	Green	100%	100%	100%	100%	100%
		Yellow	0%	0%	0%	0%	0%
		Red	0%	0%	0%	0%	0%
	PoF (Proportion of Failure)	Accept	5%	4%	0%	0%	3%
		Reject	95%	96%	100%	100%	97%
	CCI (Christoffersen's Interval)	Accept	97%	96%	100%	100%	97%
		Reject	3%	4%	0%	0%	3%
	TUFF (Time until First Failure)	Accept	4%	100%	2%	0%	100%
		Reject	96%	0%	98%	100%	0%
CBL	TL (Traffic Light)	Green	100%	96%	100%	100%	100%
		Yellow	0%	4%	0%	0%	0%
		Red	0%	0%	0%	0%	0%
	PoF (Proportion of Failure)	Accept	4%	5%	1%	0%	0%
		Reject	96%	95%	99%	100%	100%
	CCI (Christoffersen's Interval)	Accept	96%	100%	100%	100%	96%
		Reject	4%	0%	0%	0%	4%
	TUFF (Time until First Failure)	Accept	5%	95%	0%	0%	97%
		Reject	95%	5%	100%	100%	3%
IBBL	TL (Traffic Light)	Green	99%	95%	100%	100%	100%
		Yellow	1%	5%	0%	0%	0%
		Red	0%	0%	0%	0%	0%
	PoF (Proportion of Failure)	Accept	0%	0%	0%	0%	4%
		Reject	100%	100%	100%	100%	96%
	CCI (Christoffersen's Interval)	Accept	97%	100%	100%	100%	100%
		Reject	3%	0%	0%	0%	0%
	TUFF (Time until First Failure)	Accept	4%	13%	0%	0%	5%
		Reject	96%	87%	100%	100%	95%
Bank Asia	TL (Traffic Light)	Green	95%	65%	99%	100%	92%
		Yellow	5%	25%	1%	0%	7%
		Red	0%	10%	1%	0%	1%
	PoF (Proportion of Failure)	Accept	0%	2%	0%	0%	0%
		Reject	100%	98%	100%	100%	100%
	CCI (Christoffersen's Interval)	Accept	100%	100%	100%	100%	100%
		Reject	0%	0%	0%	0%	0%
	TUFF (Time until First Failure)	Accept	0%	0%	0%	0%	0%
		Reject	100%	100%	100%	100%	100%

Source: authors' estimations based on Matlab (2020a) Output

Now, the parameters estimated with GARCH (1, 1) and GARCH-t (1, 1) model for forecasting conditional volatility to estimate VaR measures of all six commercial banks are reported below:

Table 4: Estimation of Parameters from GARCH and t-GARCH approach of six Banks

Bank ID	GARCH Model				GARCH-t Model				
	Constant	Coefficients (GARCH)	Coefficients (ARCH)	K	Constant	Coefficients (GARCH)	Coefficients (ARCH)	K	DoF
PBL	0.0170	1.416e-22	1	.0013	-0.0012	0.65579	0.31912	1.266e-04	2.88
IBBL	-.0210	0	1	.0020	-0.0015	0.5448	0.4552	1.3495e-04	3.32
MTBL	0.0025	0.8390	0.1610	3.05e-04	-7.530e-04	0.4869	0.5131	1.507e-04	3.13
CBL	-.0198	0	1	0.003	-.0018	0.4871	0.4912	2.6103e-04	2.97
Al-Arafa	-.0051	0	1	5.63e-04	-.0014	0.4158	0.5842	2.05e-04	2.75
B.Asia	.0051	0.791	0.2082	7.2714e-04	-.001	0.4596	0.4566	1.7160e-04	3.17

DoF = Degrees of Freedom,

Note: The ARCH coefficients estimated under GARCH model for PBL, IBBL, CBL and Al-Arafa Bank Ltd are found 1 due to long repetitions of same log returns from repeated share price in time series amid idiosyncratic period so that the optimization of GARCH model wouldn't be able to estimate the coefficients accordingly.

Source: authors' estimations based on Matlab (2020a) Output

Table 05 reported in the Appendix is revealing the codes estimating expected shortfall (ES) for IBBL and other banks considering all these five models such as HS, HYB, GARCH, GARCH-t and RM model under rolling window method like forecasting VaR mentioned earlier for IBBL and other corresponding banks. The ES alpha was 0.025 or 2.5% as per BASEL accord III and it has been estimated with an integral of VaR measures of these five approaches that can be approximated as a sum of different VaR levels estimated with any of these five approaches in accordance with following equation as described in methodology segment:

$$\boldsymbol{ES_{\propto}(X) = \frac{1}{\propto}\int_0^{\propto} VaR_u(X)du \approx \frac{1}{N}\sum_{k=1}^{N} VaR_{\frac{k\propto}{N}}(X) \ldots\ldots (18)}$$

For Expected Shortfall estimations of all these six commercial banks mentioned earlier, we have adopted all these five models including HS, HYB, GARCH, GARCH-t and RM model to reveal a comparative scenario of estimated ES as well as VaR of the same out-sampled period. It is mentionable that ES forecasts are fitted better than VaR measures compared to the 1230 out-sample P&L outcomes or log returns also supported by simona rocioletti (2015) and Johan Engvall (2016) as depicted from the following figures:

For IBBL in the figure 07, The GARCH-t and GARCH models display obvious changes in ES estimates during periods of augmented market volatility, aligning with spikes in the P&L data. The Risk Metrics and hybrid models demonstrate smoother, less reactive estimates.

In figure 08 for Al-Arafa Bank, The ES forecasts from the GARCH-t model show high peaks during market stress, whereas the Risk Metrics model remains relatively stable. The hybrid model provides a balance between stability and reactivity.

For Bank Asia and the City Bank, these figures exhibit a similar trend where GARCH and GARCH-t models show more variability and higher ES levels during market turbulence. The Risk Metrics model follows a consistent path with minimal fluctuations.

For Prime and MTBL, The GARCH and GARCH-t models yield ES estimates that reflect higher risk during volatile periods, as indicated by larger deviations in the P&L. The HS and hybrid models provide more moderate estimates, with the hybrid model adapting more to recent changes as reflected from figure 11 and 12.

Fig 7: Estimation of ES for IBBL

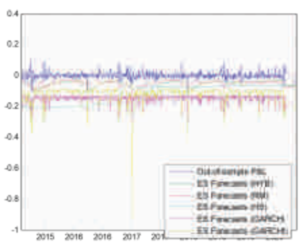


Fig 8: Estimation of ES for Al-Arafa Bank

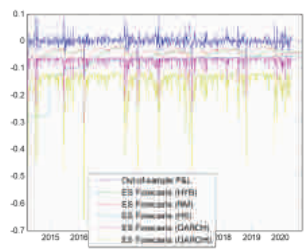


Fig 9: Estimation of ES for Bank Asia

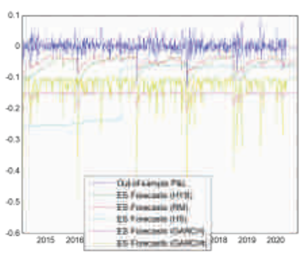


Fig 10: Estimation of ES for The City Bank

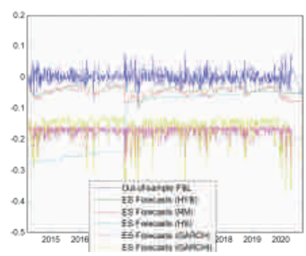


Fig 11: Estimation of ES for Prime Bank

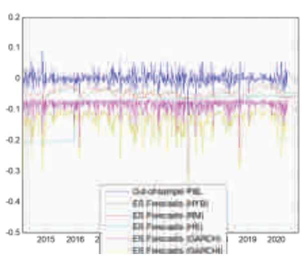
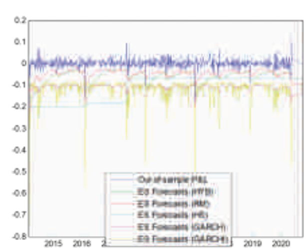


Fig 12: Estimation of ES for MTBL



Note: The above figures consist of six subplots that show the estimation of Expected Shortfall (ES) forecasts for various banks: IBBL, Al-Arafa Bank, Bank Asia, The City Bank, Prime Bank, and MTBL. The models represented include Historical Simulation (HS), Weighted Historical Simulation (Hybrid model), Risk Metrics, GARCH, and GARCH-t, with the out-of-sample P&L series for reference.

So, The estimated ES with all these five models such as HS, HYB, GARCH, GARCH-t and RM model for all these six commercial banks predicts better than earlier VaR measures as there are fewer violations observed compared to actual log returns or P&L for out-sample period. This statement can be further clarified observing the outcomes of back-testing for all these models applied to estimate ES for these six banks during the out-sampled periods as all these five models are accepted under both methods of ES back-testing test statistic as mentioned below:

Table 6: Outcomes of Back-testing of all five models estimating ES forecast for Banks

Back-Testing Approaches	ES Models	VaR Level	PBL	MTBL	IBBL	Al-Arafa	CBL	Bank Asia
Unconditional Normal	HS	0.975	Accept	Accept	Accept	Accept	Accept	Accept
	HYB		Accept	Accept	Accept	Accept	Accept	Accept
	GARCH		Accept	Accept	Accept	Accept	Accept	Accept
	GARCH-t		Accept	Accept	Accept	Accept	Accept	Accept
	RM		Accept	Accept	Accept	Accept	Accept	Accept
Unconditional-t	HS	0.975	Accept	Accept	Accept	Accept	Accept	Accept
	HYB		Accept	Accept	Accept	Accept	Accept	Accept
	GARCH		Accept	Accept	Accept	Accept	Accept	Accept
	GARCH-t		Accept	Accept	Accept	Accept	Accept	Accept
	RM		Accept	Accept	Accept	Accept	Accept	Accept

Source: authors' estimation based on Matlab (2020a) Output

It's also mentionable that adopting 2.5% alpha as per BASEL capital accord III requires 100 times rolling estimation of Expected Shortfall (ES) forecasts for 1230 out-sampled daily P&L considering all these five models using 1230 in sample observations. These rolling estimates of ES forecasts have been passed through a process of back-testing approaches each time that depicts the outcomes in a relative form as reported below:

Table 7: Relative outcomes of 100 times Back-testing of all five ES models for all Banks

Bank ID	Back-testing Approaches	Outcomes	Expected Shortfall (ES) Models				
			HS	HYB	GARCH	GARCH-t	RM
PBL	Unconditional Normal	Accept	100%	97%	100%	100%	98%
		Reject	0%	3%	0%	0%	2%
	Unconditional-t	Accept	100%	97%	100%	100%	97%
		Reject	0%	3%	0%	0%	3%
CBL	Unconditional Normal	Accept	100%	96%	100%	100%	100%
		Reject	0%	4%	0%	0%	0%
	Unconditional-t	Accept	100%	97%	100%	100%	100%
		Reject	0%	3%	0%	0%	0%

MTBL	Unconditional Normal	Accept	97%	98%	100%	100%	97%
		Reject	3%	2%	0%	0%	3%
	Unconditional-t	Accept	98%	97%	100%	100%	97%
		Reject	2%	3%	0%	0%	3%
Bank-Asia	Unconditional Normal	Accept	100%	98%	100%	100%	100%
		Reject	0%	2%	0%	0%	0%
	Unconditional-t	Accept	100%	98%	100%	100%	100%
		Reject	0%	2%	0%	0%	0%
IBBL	Unconditional Normal	Accept	100%	95%	100%	100%	100%
		Reject	0%	5%	0%	0%	0%
	Unconditional-t	Accept	100%	94%	100%	100%	100%
		Reject	0%	6%	0%	0%	0%
Al-Arafa	Unconditional Normal	Accept	98%	96%	100%	100%	97%
		Reject	2%	4%	0%	0%	3%
	Unconditional-t	Accept	97%	96%	100%	100%	97%
		Reject	3%	4%	0%	0%	3%

Source: authors' contribution based on Matlab (2020a) Output

4. Conclusion with policy implication

This paper has accomplished the objective of investigating the performance of the Hybrid model for the first time on the Banking sector in Bangladesh. This is the first application of this parsimonious model forecasting expected shortfall (ES) risk measure on a particularly noisy out-sample period's P&L outcomes of six leading commercial banks in Bangladesh. Our investigation reveals that this user-friendly Hybrid model, forecasting value at risk (VaR) as well as expected shortfall (ES) for six commercial banks, performs poorly compared to other more advanced models such as GARCH and GARCH-t as reflected in the outcomes of the back-tests. This is despite the fact that our sample period by design considers one of the most idiosyncratic market conditions as reflected by their idiosyncratic share price movements on DSE. Moreover, this model performs poorly in case of estimating VaR compared to RM and HS models which are relatively less complicated (more user friendly), as per the outcomes of the back-tests. As a consequence, the investors should not be illuded by this Hybrid model when they are required to investigate the market risk of their investment in banking sector. This is as per the back-tests recommended by the BASEL Accord-III for Expected Shortfall. Although this hybrid model for forecasting VaR and Expected Shortfall (ES) is recommended (Boudoukh, Richardson and Whitelaw, 2007) as empirically attractive alternative, in this investigation we find that the attraction of parsimonious modeling with Hybrid model is a myth rather than a reality. Moreover, investors should adopt GARCH and GARCH-t models to estimate and forecast the market risk of their investment when they prefer using VaR as their risk measure. Also, when investors prefer using ES as their risk measure, all the four alternatives RM, HS, GARCH and GARCH-t are found more useful than the Hybrid model. In addition, commercial banks are recommended to use these four models to estimate the maximum loss with certain confidence level under adverse movements of markets such as interest rate, foreign exchange, and capital market index while constructing their trading portfolios consisting of fixed income security, foreign exchange position and equity position.

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Appendix

Table 05: Matlab Codes for estimating expected shortfall (ES) for IBBL using all models

```
function [ res ] = ES_forecast(r,alpha,outofsample_size, bandwidth_percent,lambda )

ESalpha = sort(linspace(0.0001,alpha,100),'descend');
SUM_HS=0;
SUM_HYB=0;
SUM_RM=0;
SUM_GARCH=0;
SUM_GARCHt=0;

Inonan_HS=0;
Inonan_HYB=0;
Inonan_RM=0;
Inonan_GARCH=0;
Inonan_GARCHt=0;

for i=1:length(ESalpha)
   res=VaR_forecasts(r,ESalpha(i),outofsample_size,bandwidth_percent,lambda);
if isnan(res.VaR_HS)~=1;  SUM_HS=SUM_HS+res.VaR_HS; Inonan_HS=Inonan_HS+1; end
if isnan(res.VaR_HYB)~=1; SUM_HYB=SUM_HYB+res.VaR_HYB; Inonan_HYB=Inonan_HYB+1; end
if isnan(res.VaR_RM)~=1; SUM_RM=SUM_RM+res.VaR_RM; Inonan_RM=Inonan_RM+1; end
if isnan(res.VaR_GARCH)~=1; SUM_GARCH=SUM_GARCH+res.VaR_GARCH; Inonan_
GARCH=Inonan_GARCH+1; end
if isnan(res.VaR_GARCH_t)~=1; SUM_GARCHt=SUM_GARCHt+res.VaR_GARCH_t; Inonan_
GARCHt=Inonan_GARCHt+1; end
end
res.ES_HS=SUM_HS/Inonan_HS;
res.ES_HYB=SUM_HYB/Inonan_HYB;
res.ES_RM=SUM_RM/Inonan_RM;
res.ES_GARCH=SUM_GARCH/Inonan_GARCH;
res.ES_GARCH_t=SUM_GARCHt/Inonan_GARCHt;
end
```

Source: authors' estimations